SPAN-WISE FLUCTUATING MHD CONVECTIVE FLOW OF WALTERS LIQUID (MODEL B') FLUID THROUGH POROUS MEDIUM IN A VERTICAL POROUS CHANNEL WITH THERMAL RADIATION AND HEAT SOURCE

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Abstract: Exact solution of an unsteady magneto-hydrodynamic (MHD) convective flow problem of an incompressible, electrically conducting and visco-elastic (second order) fluid through a porous medium bounded by two infinite vertical porous plates is obtained analytically. The fluid is injected with constant velocity through the channel wall at $y^* = -\frac{d}{2}$ and simultaneously removed with same velocity through the other wall at $y^* = +\frac{d}{2}$. The temperature of the plate at $y^* = +\frac{d}{2}$ is assumed to be fluctuating span-wise cosinusoidally as $T^*(y^*, z^*, t^*) = T_1 + (T_2 - T_1)\cos\left(\frac{\pi z^*}{d} - \omega^* t^*\right)$. A magnetic field of uniform strength is applied perpendicular to the planes of the channel plates. The magnetic Reynolds number is assumed very small so that the induced magnetic field is neglected. The temperature difference between the plates is high enough to induce the heat due to radiation. The Rosseland approximation is used to describe the radiation heat flux for the fluid as optically-thick gray gas, absorbing/emitting but non-scattering medium. Exact solution of the partial differential equations governing the flow under the prescribed boundary conditions has been obtained for the velocity and the temperature fields. The velocity, temperature and the skin-friction and Nusselt number in terms of their amplitudes and phase angles have been shown graphically to observe the effects of viscoelastic parameter γ, injection/suction parameter λ, Grashof number Gr, Hartmann number M, the permeability of the porous medium K, Prandtl number P_r, radiation parameter N, pressure gradient A and the frequency of oscillation ω. The final results are then discussed in detail in the last section of the paper with the help of figures.

Keywords: Magnetohydromagnetic (MHD), Convective, Span-Wise Fluctuating, Viscoelastic, Porous Medium, Radiation.

Introduction: Many common liquids such as oils, certain paints, polymer solution, some organic liquids and many new material of industrial importance exhibit both viscous and elastic properties. Therefore, these fluids called viscoelastic fluids are being studied extensively. Many researchers have shown their interest in the fluctuating flow of a viscous incompressible fluid past an infinite or semi-infinite flat plate. Viscoelastic fluid flow through porous media has attracted the attention of scientists and engineers because of its importance notably in the flow of oil through porous rocks, the extraction of energy from geothermal region and drug permeation through human skin. The knowledge of flow through porous media is useful in the recovery of crude oil efficiently from the pores of reservoir rocks by displacement with immiscible water. The flow through porous media occurs in the ground water hydrology, irrigation, and drainage problems and also in absorption and filtration processes in chemical engineering. The scientific treatment of the problem of irrigation, soil erosion and tile drainage are the present developments of porous media. Nakayama and Koyama [1] studied buoyancy induced flow of a non-Newtonian fluid over a non-isothermal body of arbitrary shape in a fluid saturated porous medium. Ariel [2] analyzed the flow of viscoelastic fluid past a porous plate. MHD flow of a viscoelastic fluid past a stretching surface was studied by Andersson [3]. Pillai et al. [4] analyzed viscoelastic boundary layer flow through porous medium with heat transfer. Sharma and Pareek [5],[6] examined an Unsteady flow and heat transfer through an elastico-viscous liquid along an infinite hot vertical porous moving plate in different situation. Rahman and Sarkar [7] investigated the unsteady MHD flow of a viscoelastic Oldroyd fluid under time varying body forces through a rectangular channel. Singh and Singh [8] studied an

MHD flow of a dusty viscoelastic (Oldroyd B-liquid) through a porous medium between two parallel plates inclined to horizon. Datti et al [9] studied MHD viscoelastic fluid flow over anon-isothermal stretching sheet. Roy and Chaudhury [10] analyzed heat transfer by laminar flow of an elastico-viscous liquid along a plane wall with periodic suction. Hameed and Nadeem [11] studied unsteady MHD flow of a non-Newtonian fluid on a porous plate.

Hayat et al [12] discussed periodic unsteady flows of a non-Newtonian fluid. Kumar and Sivaraj [13] studied MHD mixed convective viscoelastic fluid flow in a permeable vertical channel with Dufour, effect and chemical reaction. Singh [14] analyzed viscoelastic mixed convection MHD oscillatory flow through a porous medium filled in a vertical channel. Singh [15] analyzed an oscillatory mixed convection flow of a viscoelastic electrically conducting fluid in an infinite vertical channel filled with porous medium. Considering the Hall effects Attia [16] discussed unsteady Hartmann flow of a viscoelastic fluid. Attia [17] analyzed Unsteady MHD Couette flow of a viscoelastic fluid with heat transfer.Alphonsa and Singh [18] discussed Hall effect on radiating span-wise fluctuating MHD convective flow through porous medium. Damseh and Shannak [19] Visco-elastic fluid flow past an infinite vertical porous plate in the presence of first order chemical reaction. Sivaraj and Rushi Kumar [20] studied MHD mixed convective flow of viscoelastic and viscous fluids in a vertical porous channel. Misra et al [21] studied Hydromagnetic flow and heat transfer of a second-grade viscoelastic fluid in a channel with oscillatory stretching walls: application to the dynamics of blood flow also Choudhury and Das [22] analyzed visco-elastic MHD free convective flow through porous media in presence of radiation and chemical reaction with heat and mass transfer. Recently Gorla et al [23] analyzed the effect of unsteady heat and mass transfer in MHD viscoelastic fluid flow through porous medium between two inclined porous parallel plates with soret effect and G-Jitter force.

The objective of the present paper is to study an unsteady MHD convective flow of a viscoelastic (Walter's liquid-B) fluid through a porous medium filled in a vertical channel in the presence of heat source. Constant injection and suction is applied at the left and the right infinite porous plates respectively. A uniform magnetic field is applied along the axis perpendicular to the planes of the plates. The magnetic Reynolds number is assumed very small so that the induced magnetic field is neglected. The temperature difference between the plates of the channel is sufficiently high to induce heat radiation. An exact solution of the partial differential equations governing the flow problem is obtained and the effects of various flow parameters on the velocity field and the skin friction are discussed in the last section of the paper with the help of figures.The object of the present paper is to study an unsteady MHD convective flow of a viscoelastic (Walter's liquid-B) fluid through a porous medium filled in a vertical channel in the presence of heat source. Constant injection and suction is applied at the left and the right infinite porous plates respectively. A uniform magnetic field is applied along the axis perpendicular to the planes of the plates. The magnetic Reynolds number is assumed very small so that the induced magnetic field is neglected. The temperature difference between the plates of the channel is sufficiently high to induce heat radiation. An exact solution of the partial differential equations governing the flow problem is obtained and the effects of various flow parameters on the velocity field and the skin friction are discussed in the last section of the paper with the help of figures.

Mathematical Analysis: An oscillatory MHD convective flow of a Walters liquid Model B' (viscoelastic), incompressible and electrically conducting fluid through a porous medium in a vertical channel is considered. The constitutive equations for the rheological equation of state for the viscoelastic fluid (Walters liquid Model B') are

$$p_{ik} = -pg_{ik} + p^*_{ik} \quad (1)$$

$$p^*_{ik} = 2\int_{-\infty}^{t} \psi(t-t^*)\, e^{(1)}_{ik}(t^*)dt^* . \quad (2)$$

Where in $\psi(t-t^*) = \int_{-\infty}^{t} \frac{N(\tau)}{\tau} e^{-[(t-t^*)/\tau]} d\tau$, and $N(\tau)$ is distribution function of relaxation time τ. In the above equation p_{ik} is the stress tensor, p is an arbitrary isotropic pressure, $g_{ik}$is the metric tensor of a fixed co-ordinate system xi and $e^{(1)}_{ik}$ is is the rate of strain tensor. It was shown by Walter (1964) that equation (2) can be put in the following generalized form which is valid for all types of motion and stress

$$p^*_{ik}(x,t) = 2\int_{-\infty}^{t} \psi(t-t^*)\, \frac{\partial x^i}{\partial x^{*m}} \frac{\partial x^k}{\partial x^{*r}} e^{(1)mr}(x^*,t^*)dt^*. \quad (3)$$

Wherein x^{*i} is the position at times $t^*$of the element which is instantaneously at the point $x^{\,i}$at the time t. The fluid with the equation (1) to (3) has been designated as the liquid B'. In the case of the liquid with short memories i.e. short relaxation times, the above equation can be written in the following simplified form:

$p_{ik}^*(x,t) = 2\eta_0 e^{(1)ik} - 2k_0 \frac{\partial e^{(1)ik}}{\partial t}$,
(4)
where $\eta_0 = \int_0^\infty N(\tau)d\tau$ is limiting viscosity at the small rates of shear, $k_0 = \int_0^\infty \tau N(\tau)d\tau$ and $\frac{\partial}{\partial t}$ denotes the convected time derivative.

The insulated plates of the channel are at distance 'd' apart. The porous walls of the vertical channel are lying in the $y^* = \pm\frac{d}{2}$ planes and the fluid is injected through the left porous plate with constant velocity (V) and simultaneously sucked through the other plate with the same velocity (V). The x^*- axis is oriented vertically upwards along the centreline of the channel. The y^*-axis taken perpendicular to the planes of the plates and a transverse magnetic field of uniform strength $\vec{B} = (0, B_0, 0)$ is applied along this axis. The non-uniform temperature of the plate at $y^* = +\frac{d}{2}$ is assumed to be varying span-wise cosinusoidally in space and time both as $T^*(y^*, z^*, t^*) = T_1 + (T_2 - T_1)\cos\left(\frac{\pi z^*}{d} - \omega^* t^*\right)$. (5)

Since the plates of the channel are of infinite extent in the x^*direction, therefore, all the physical quantities except the pressure are independent of x^*. All fluid properties are assumed constant except variation of density with temperature only in the body force term. The equation of continuity $\nabla.\vec{V} = 0$ for the constant injection/suction at the channel plates integrates to $v *= V$ where $\vec{V} = (u^*, v^*, 0)$ represents the velocity components in the directions (x^*, y^*, z^*) respectively. The physical configuration of the problem is shown in Figs. 1a & 1b.

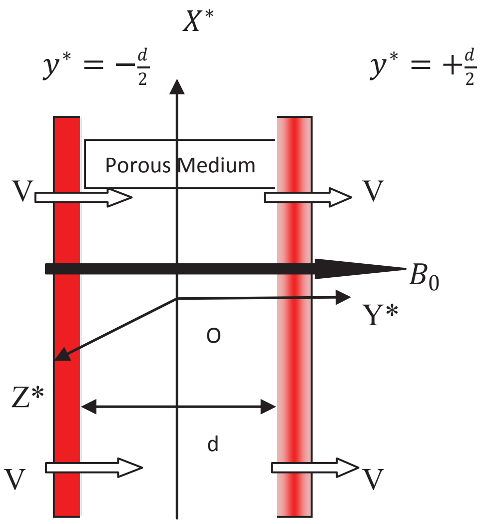

Fig.1a. Hot Vertical Channel

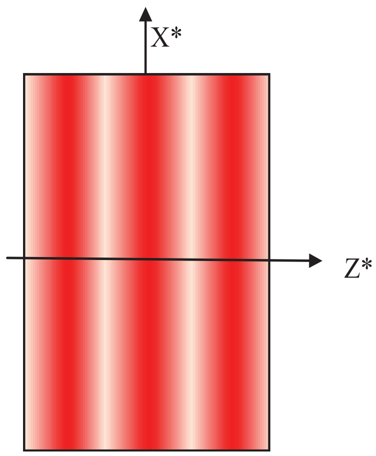

Fig.1b. Span-Wise Cosinusoidal Plate Temperature

Following Attia and Ewis [24] and Kumar and Chand [25] and taking into account the usual Boussinsq's approximation the magnetohydrodynamic (MHD) mixed convection flow in the vertical channel is governed by the following momentum and energy differential equations:

$$\frac{\partial u^*}{\partial t^*} + V\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta_1 \nabla^2 u^* + \vartheta_2 \nabla^2\left(\frac{\partial u^*}{\partial t^*}\right) - \frac{\sigma B_0^2}{\rho}u^* - \frac{\vartheta_1}{K^*}u^* + g\beta(T^* - T_1), \quad (6)$$

$$\rho c_p\left(\frac{\partial T^*}{\partial t^*} + V\frac{\partial T^*}{\partial y^*}\right) = k\nabla^2 T^* - \frac{\partial q^*}{\partial y^*} - Q^*(T^* - T_1), \quad (7)$$

where $\nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

The heat flux due to radiation and for an optically thick gray gas is expressed by using Rosseland approximation as

$$q^* = -\frac{4\sigma^*}{3k^*}\frac{\partial T^{*4}}{\partial y^*}, \quad (8)$$

We assume that the temperature differences within the flow are sufficiently small such that $T*^4$ may be expanding in Taylor series about T_1. Neglecting higher order terms and retaining first term only, we obtain

$$T^{*4} \cong 4T_1^3 T^* - 3T_1^4. \quad (9)$$

Substituting (9) into (8) and simplifying, we obtain

$$\frac{\partial q^*}{\partial y^*} = -\frac{16\sigma^* T_1^3}{3k^*}\frac{\partial^2 T^*}{\partial y^{*2}}. \qquad (10)$$

The substitution of equation (10) into the energy equation (7) for the heat due to radiation, we get

$$\rho c_p \left(\frac{\partial T^*}{\partial t^*} + V\frac{\partial T^*}{\partial y^*}\right) = k\nabla^2 T^* + \frac{16\sigma^* T_1^3}{3k^*}\frac{\partial^2 T^*}{\partial y^{*2}} - Q^*(T^* - T_1), \qquad (11)$$

The boundary conditions for the problem are

$$y^* = -\frac{d}{2}: \quad u^* = 0,\ T^* = T_1, \qquad (12)$$

$$y^* = \frac{d}{2}: \quad u^* = 0,\ T^* = T_1 + (T_2 - T_1)\cos\left(\frac{\pi z^*}{d} - \omega^* t^*\right). \qquad (13)$$

Introducing the following non-dimensional quantities

$$x, y, z = \frac{x^*, y^*, z^*}{d}, t = \omega^* t^*, \omega = \frac{\omega^* d^2}{\vartheta_1}\ u = \frac{u^*}{V}, \theta = \frac{T^* - T_1}{T_2 - T_1},\ p = \frac{p^*}{\rho V^2}, \qquad (14)$$

into equations (6) and (11) we get

$$\omega\frac{\partial u}{\partial t} + \lambda\frac{\partial u}{\partial y} = -\lambda\frac{\partial p}{\partial x} + \nabla^2 u + \gamma\omega\nabla^2\left(\frac{\partial u}{\partial t}\right) - (M^2 + K^{-1})u + Gr\,\theta, \qquad (15)$$

$$\omega Pr\frac{\partial \theta}{\partial t} + \lambda Pr\frac{\partial \theta}{\partial y} = \nabla^2\theta + \frac{4}{3N}\frac{\partial^2\theta}{\partial y^2} - S\theta, \qquad (16)$$

where $\gamma = \frac{\vartheta_2}{d^2}$ is the viscoelastic parameter, $\lambda = \frac{Vd}{\vartheta_1}$ is the injection/suction parameter,
$Gr = \frac{g\beta d^2(T_2 - T_1)}{\vartheta V}$ is the Grashof number, $M = B_0 d\sqrt{\frac{\sigma}{\rho\vartheta_1}}$ is the Hartmann number,
$K = \frac{K^*}{d^2}$ is the permeability of the porous medium, $Pr = \frac{\mu c_p}{k}$ is the Prandtl number,
$N = \frac{kk^*}{4\sigma^* T_1^3}$ is the radiation parameter, $S = \frac{Q^* d^2}{k}$ is the heat source.

The boundary conditions in the dimensionless form become

$$y = -\frac{1}{2}: u = 0,\ \theta = 0, \qquad (17)$$

$$y = \frac{1}{2}: u = 0,\ \theta = \cos(\pi z - t). \qquad (18)$$

Solution of the Problem: In order to obtain the solution of this flow in the porous channel when the fluid is acted upon by an unsteady periodic drop in pressure, we assume the solution in complex variable notations as

$$u(y, z, t) = u_0(y)e^{i(\pi z - t)},\ \theta(y, z, t) = \theta_0(y)e^{i(\pi z - t)},\ -\frac{\partial p}{\partial x} = Ae^{i(\pi z - t)}, \qquad (19)$$

where A is a constant. The real part of the solution will have physical significance.
The boundary conditions (17) and (18) can also be written in complex notations as

$$y = -\frac{1}{2}: u = 0,\ \theta = 0, \qquad (20)$$

$$y = \frac{1}{2}: u = 0,\ \theta = e^{i(\pi z - t)}. \qquad (21)$$

Substituting equation (19) into equations (15) and (16) we obtain following equations

$$(1 - i\omega\gamma)u_0'' - \lambda u_0' - \{\pi^2 + M^2 + K^{-1} - i\omega(1 + \pi^2\gamma)\}u_0 = -\lambda A - Gr\theta_0, \qquad (22)$$

$$\left(1 + \frac{4}{3N}\right)\theta_0'' - \lambda Pr\theta_0' - (\pi^2 + S - i\omega\lambda Pr)\theta_0 = 0, \qquad (23)$$

where the primes in these ordinary differential equations denote differentiation with respect to y.

The boundary conditions (20) and (21) reduce to

$$y = -\frac{1}{2}:\ u_0 = 0,\quad \theta_0 = 0, \qquad (24)$$

$$y = \frac{1}{2}:\ u_0 = 0,\quad \theta_0 = 1. \qquad (25)$$

The solution of equation (22) for the velocity field under the boundary conditions (24) and (25) is obtained as

$$\mathrm{u}(y, z, t) = \left[\begin{array}{c} \frac{\lambda A}{l}\left\{1 + \frac{e^{my}\sinh\frac{n}{2} - e^{ny}\sinh\frac{m}{2}}{\sinh\left(\frac{m-n}{2}\right)}\right\} - \frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left(\frac{e^{ry-\frac{s}{2}}}{C_1} - \frac{e^{sy-\frac{r}{2}}}{C_2}\right) + \\ \frac{Gr}{4\sinh\left(\frac{m-n}{2}\right)\sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{c}\left(\frac{e^{\frac{r-s}{2}}}{C_1} - \frac{e^{-\frac{r-s}{2}}}{C_2}\right)\left(e^{my-\frac{n}{2}} - e^{ny-\frac{m}{2}}\right) \\ + \left(\frac{C_1 - C_2}{C_1 C_2}\right)\left(e^{my+\frac{n}{2}} - e^{ny+\frac{m}{2}}\right)e^{-\frac{\lambda Pr}{2\left(1+\frac{4}{3N}\right)}}\end{array}\right\}\end{array}\right]e^{i(\pi z - t)} \qquad (26)$$

where $l = [\pi^2 + M^2 + K^{-1} - i\omega(1 + \gamma\pi^2)]$ $C_1 = (1 - i\omega\gamma)r^2 - \lambda r - l, C_2 = (1 - i\omega\gamma)s^2 - \lambda s - l,$ $m = \frac{\lambda+\sqrt{\lambda^2+4l(1-i\omega\gamma)}}{2(1-i\omega\gamma)}$, $n = \frac{\lambda-\sqrt{\lambda^2+4l(1-i\omega\gamma)}}{2(1-i\omega\gamma)}$,

$r = \frac{\lambda Pr+\sqrt{\lambda^2 Pr^2+4\left(1+\frac{4}{3N}\right)(\pi^2+S-i\omega Pr)}}{2\left(1+\frac{4}{3N}\right)}$, $s = \frac{\lambda Pr-\sqrt{\lambda^2 Pr^2+4\left(1+\frac{4}{3N}\right)(\pi^2+S-i\omega Pr)}}{2\left(1+\frac{4}{3N}\right)}$.

Similarly, the solution of equation (23) for the temperature field under the boundary conditions (24) and (25) is obtained as

$$\theta(y,z,t) = \left(\frac{e^{ry-\frac{s}{2}}-e^{sy-\frac{r}{2}}}{2\sinh\left(\frac{r-s}{2}\right)}\right)e^{i(\pi z-t)} . \tag{27}$$

The amplitude is $|F| = \sqrt{F_r^2 + F_i^2}$ and the phase angle $\varphi = \tan^{-1}\frac{F_i}{F_r}$, (28)

wherein

$$F = F_r + i\,F_i = \left[\begin{array}{l}\frac{\lambda A}{l}\left(\frac{me^{-\frac{m}{2}}\sinh\frac{n}{2}-ne^{-\frac{n}{2}}\sinh\frac{m}{2}}{\sinh\left(\frac{m-n}{2}\right)}\right)-\frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left(\frac{r}{C_1}-\frac{s}{C_2}\right)e^{-\frac{\lambda Pr}{2\left(1+\frac{4}{3N}\right)}}+\\ \frac{Gr}{4\sinh\left(\frac{m-n}{2}\right)\sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{l}\left(\frac{e^{\frac{r-s}{2}}}{C_1}-\frac{e^{-\frac{r-s}{2}}}{C_2}\right)(m-n)e^{-\frac{\lambda}{2(1-i\omega\gamma)}}+\\ \left(\frac{C_1-C_2}{C_1C_2}\right)\left(me^{-\frac{m-n}{2}}-ne^{\frac{m-n}{2}}\right)e^{-\frac{\lambda Pr}{2\left(1+\frac{4}{3N}\right)}}\end{array}\right\}\end{array}\right] \tag{29}$$

Similarity, we can get the Nusselt number, Nu, the heat transfer coefficient in terms of its amplitude $|H|$ and the phase angle ψ from equation (27) for the temperature field as

$$Nu = |H|\ \cos(\pi z - t + \psi), \tag{30}$$

With $|H| = H_r + i\,H_i = \frac{(r-s)e^{-\frac{\lambda Pr}{2\left(1+\frac{4}{3N}\right)}}}{2\sinh\left(\frac{r-s}{2}\right)}$, (31)

where the amplitude $|H|$ and the phase angle ψ of the rate of heat transfer are given as

$$|H| = \sqrt{H_r^2 + H_i^2}, \qquad \psi = \tan^{-1}\frac{H_i}{H_r} . \tag{32}$$

Results and Discussion: An exact solution of an unsteady MHD convective flow of Walters liquid Model B′ (viscoelastic) through porous medium in a vertical porous channel is obtained in the presence of a heat source. The plate temperature of the channel varies span-wise cosinusoidally. The two porous plates are subjected to constant injection and suction. It is also assumed that the conducting fluid is optically-thick gray gas, absorbing/ emitting radiation and non-scattering. The solution so obtained is evaluated numerically for different sets of values of the parameters involved in the flow field. In order to have a better insight of the influence of the parameters on the velocity and temperature fields these numerical values are then illustrated through figures. The influence of each of the parameters on the physical quantities like the velocity, the temperature, the amplitude and the phase of the skin-friction and rate of heat transfer are depicted through figures.

The effects of different parameters on the velocity field $u(y,z,t)$ are shown in Figure 2 Different curves in this figure represent the sets of various values of the parameters listed in Table 1. This figure clearly shows that the velocity is maximum in the middle of the channel which leads to parabolic velocity profiles in the channel as expected. curve I corresponds to the case of Newtonian fluid. Remaining curves are compared with the curve II to assess the influence of each parameter on the velocity. This figure clearly shows that curves IV, V, VII and XI lie above the curve II which means that the velocity increases with the increase of injection/suction parameter λ, Grashof number Gr, permeability of the porous medium K and favorable pressure gradient A respectively. There is a sharp rise in the velocity with the increase of the injection/suction parameter λ . The increase of velocity with the increase of the Grashof number Gr physically means that the enhancement of the buoyancy force leads to increase of the velocity $u(y,z,t)$. The increase of velocity with the increase of permeability of the porous medium indicates that the resistance posed by the porous medium reduces as the permeability of the medium increases because of which the velocity increases. As expected the larger favorable pressure gradient in the channel leads to faster flow, hence, velocity increases.

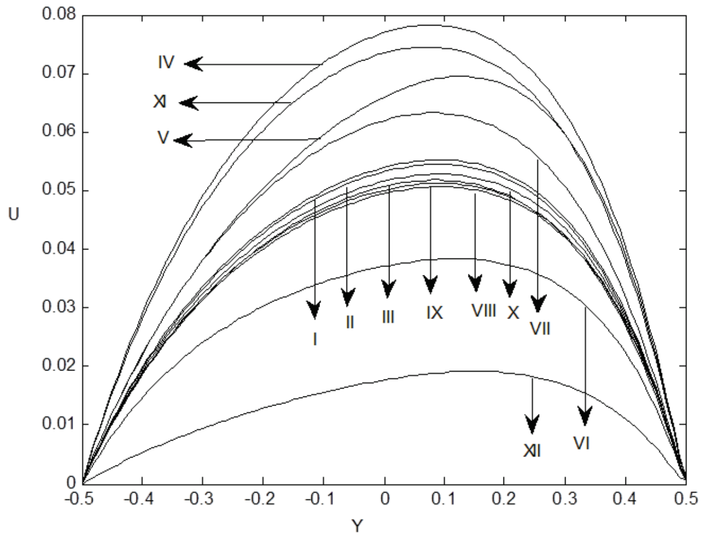

Fig. 2. Velocity Profiles for z=0.5 and t=π/2

Table1. Sets of Parameter Values Plotted in Fig. 2

γ	λ	Gr	M	K	Pr	N	S	A	ω	Curves
0	0.5	1	2	0.2	0.7	1	0.5	2	1	I
0.2	0.5	1	2	0.2	0.7	1	0.5	2	1	II
0.5	0.5	1	2	0.2	0.7	1	0.5	2	1	III
0.2	0.8	1	2	0.2	0.7	1	0.5	2	1	IV
0.2	0.5	2	2	0.2	0.7	1	0.5	2	1	V
0.2	0.5	1	4	0.2	0.7	1	0.5	2	1	VI
0.2	0.5	1	2	1.0	0.7	1	0.5	2	1	VII
0.2	0.5	1	2	0.2	7.0	1	0.5	2	1	VIII
0.2	0.5	1	2	0.2	0.7	5	0.5	2	1	IX
0.2	0.5	1	2	0.2	0.7	1	5.0	2	1	X
0.2	0.5	1	2	0.2	0.7	1	0.5	3	1	XI
0.2	0.5	1	2	0.2	0.7	1	0.5	2	5	II

The effects of other parameters like viscoelastic parameter γ, Hartmann number M, Prandtl number P_r, radiation parameter N, heat source S and frequency of oscillations ω are represented by curves III, VI, VIII, IX, X and XII respectively. From this figure it can be easily observed that these curves lie below the curve II. This means that the flow velocity decreases with the increase of these parameters. The flow retards due to increases viscoelasticity of the fluid. Lorentz force which is introduced due to the application of the transverse magnetic field retards the velocity. This force gives a dragging effect on the flow. The two values of the Prandtl number P_r=0.7 and P_r=7 are chosen to represent most common fluids air and water respectively. It is evident that the velocity is less in water than in air. Since the Prandtl number gives the relative importance of viscous dissipation to the thermal dissipation so for larger Prandtl number viscous dissipation is predominant and due to this velocity decreases. The increase of radiation N, heat source S and the frequency ω lead to a decrease in velocity.

The variation of the amplitude |F| and the phase angle ϕ of the skin-friction against ω are shown in Fig.3 and 4 respectively with the increase of different parameters like the viscoelastic parameter γ, injection/suction parameter λ, Grashof number Gr, Hartmann number M, permeability of the porous medium K, Prandtl number P_r, radiation parameter N, heat source parameter S and the pressure gradient is presented. It is obvious from figure 3 that for any set of parameters listed in Table 2 the amplitude goes on decreasing with increasing frequency of oscillationsω. The decrease is sharp in |F| for small oscillations but then reduces as ω increases further. Comparing curves III, IV, VI and X with the curve I reveals that the skin-friction amplitude increases with the increase of injection/suction parameter λ, Grashof number Gr, permeability of the porous medium K and the pressure gradient A. It is true physically also because the increase in these parameters results into velocity increase which consequently leads to the enhancement of shear stress. However, the increase in viscoelastic parameter γ, Hartmann number M, Prandtl number P_r, the radiation parameter N and heat source S represented by curves II, V, VII, VIII and IX when compared with curve I attribute towards the decrease in the amplitude of the skin-friction.

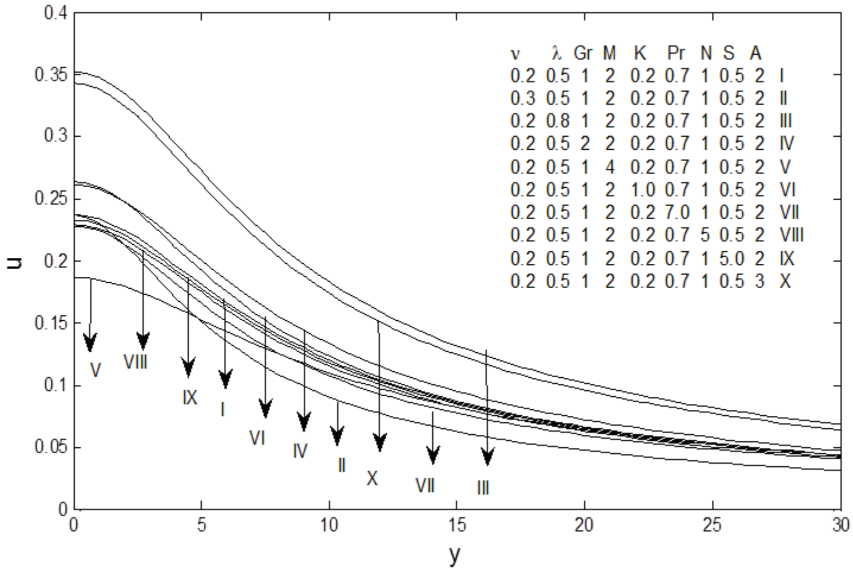

Fig. 3. Amplitude Of Skin Friction

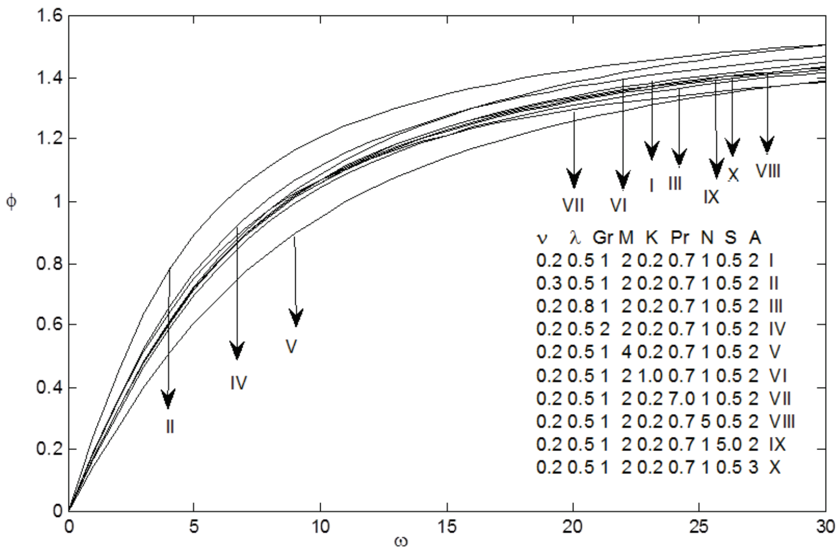

Fig.4. Phase of Skin Friction

The behavior of the phase angle φ of the skin-friction τ_L is shown in Figure 4 for different values of various sets of flow parameters. From this figure it is evident that there is always a phase lead because its values computed numerically remain positive throughout for any set of values of the flow parameters. There is almost an exponential increase of φ with increasing frequency ω for all sets of values considered. We notice by comparing curves II, III, IV, VI, VII and VIII with curve I that the phase angle increases with increasing viscoelastic parameter γ, injection/suction parameter λ, Grashof number Gr, permeability of the porous medium K, Prandtl number P_r and radiation parameter N. However, the phase lead decreases with the increase of Hartmann number M, heat source S and pressure gradient A as is indicated by the comparison of curves V, IX and X with curve I.

The variation of the temperature with the injection/suction parameter λ, Prandtl number P_r, radiation parameter N, heat source S and the frequency of oscillations ω are shown in Fig.5. It is observed from this figure that the temperature decreases with the increase of either of these parameters.

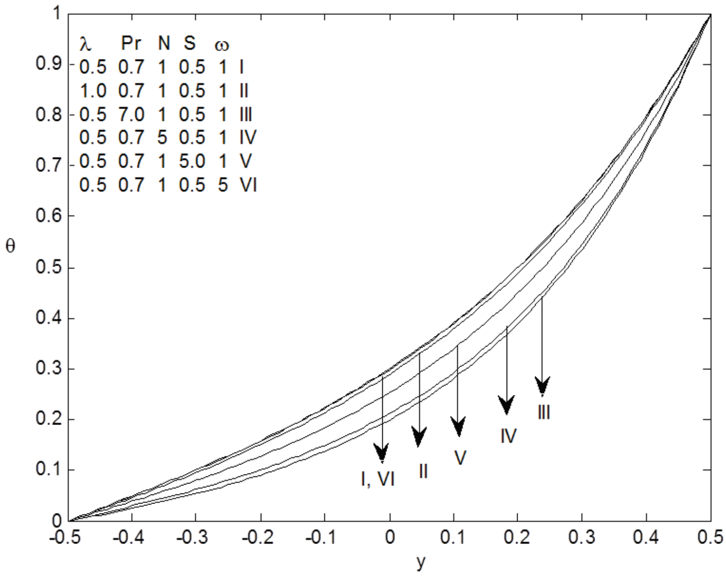

Fig. 5. Temperature Profiles for Z=0.5 and T=Π/2

The amplitude $|H|$ and the phase angle of the rate of heat transfer against the frequency of oscillations ω are illustrated in Fig. 6 and Fig.7 respectively. It is evident from Fig. 6 that the amplitude $|H|$ decreases with the increase of injection/suction parameter λ, Prandtl number P_r, the radiation parameter N and heat source S. The amplitude in the case of water (P_r =7) decreases rapidly with increasing ω and becomes negligible for large values frequency of oscillations ω. Fig.7 shows that there always remains a phase lead with the increase of injection/suction parameter λ, the radiation parameter N and heat source S and this phase lead increases linearly as ω goes on increasing. It is also notice from this figure that with the increase of Prandtl number P_r the phase starts oscillating between the phase lead and the phase lag as the frequency ω increases. Fig.8 gives a clear cut picture of various parameters like λ, N and S are also begin to oscillate between the phase lead and the phase lag as the frequency ω increases.

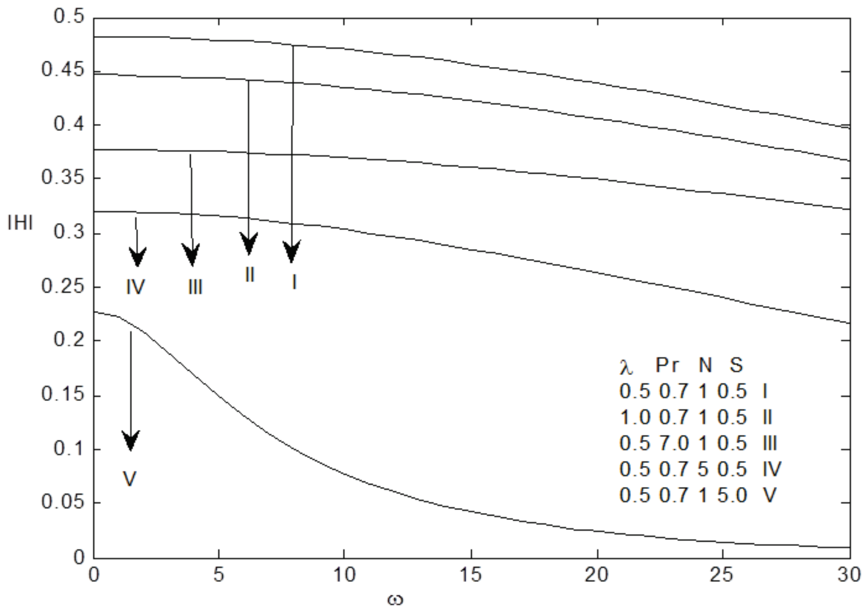

Fig. 6. Amplitude of Nusselt Number

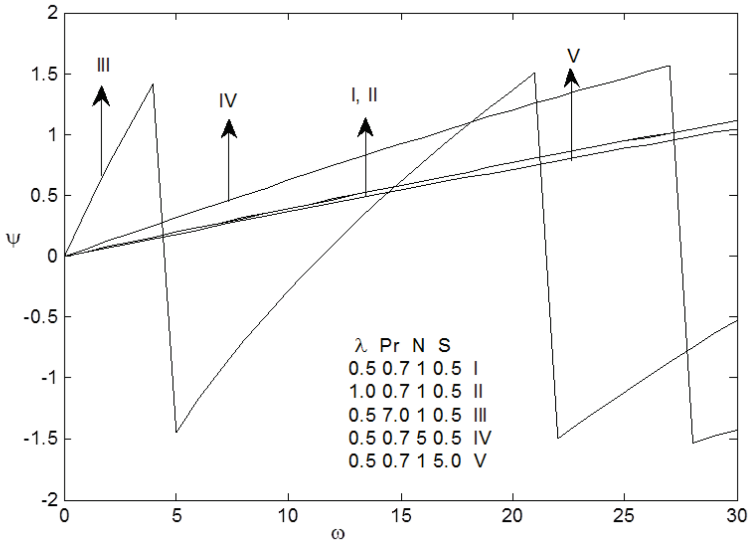

Fig. 7. Phase Angle of Nusselt Number

Conclusions: The following conclusions are made from the above discussion:

- The increase of buoyancy force leads to increase of the velocity $u(y,z,t)$.
- The increase of velocity with the increase of permeability of the porous medium indicates that the resistance posed by the porous medium reduces as the permeability of the medium increases because of which the velocity increases.
- The velocity also increases with the increase of injection/suction parameter λ and favorable pressure gradient A.
- The flow retards as the fluid viscoelasticity increases.
- Lorentz force which is introduced due to the application of the transverse magnetic field retards the velocity.
- The increase of Prandtl number, radiation N and the frequency ω leads to a decrease in velocity.
- The temperature decreases with the increase of either of the parameters involved.
- The amplitude of skin friction increases due to the increase of all those parameters because of which flow accelerates.
- It is true physically also because the increase in these parameters results into velocity increase which consequently leads to the enhancement of shear stress.
- However, the increase in Hartmann number M, Prandtl number P_r or the radiation parameter N, attribute towards the decrease in the amplitude of the skin-friction.
- There is always a phase lead of the skin friction.
- The amplitude of rate of heat transfer reduces due to the increase of all parameters involved.
- For increasing λ, N and S there is always a phase lead of rate of heat transfer and remains linear over the values of ω considered.
- However, for increasing Prandtl number phase starts oscillating between the phase lead and the phase lag as the frequency ω increases.

Nomenclature:

A	-Constant	M	-Hartmann Number
B_o	-Uniform Magnetic Field	N	-Radiation Parameter
C_p	-Specific Heat At Constant Pressure	$P*$	-Pressure
D	-Distance Between Plates	Pr	-Prandtl Number
$\|F\|$	-Amplitude Of Skin-Friction	$Q*$	-Heat Absorption Coefficient
G	-Acceleration Due To Gravity	$q*$	-Radiative Heat Flux
Gr	-Grashof Number	S	-Heat Source Parameter
$\|H\|$	-Amplitude Of Rate Of Heat Transfer	t	-Time
k	-Thermal Conductivity	T^*	-Fluid Temperature
$k*$	- The Mean Absorption Coefficient	T_1, T_2	-Constant Temperatures
K	-Porous Medium Permeability	$u*$	-Fluid Velocity In X^* -Direction

V -Injection/Suction Velocity
u, v, w -Velocity Components Along X, Y, Z-Axis
x, y, z -Variables Along X, Y, Z −Directions

Greek Symbols:

β -Coefficient Of Thermal Expansion
$\sigma *$ -Stefan Boltzmann Constant
σ -Electrical Conductivity
ρ -Fluid Density
M -Viscosity
ϑ_1 -Kinematic Coefficient Of Viscosity
ϑ_2 -Viscoelasticity
ω -Frequency Of Oscillations
φ -Phase Angle Of Skin-Friction
ψ -Phase Angle Of Heat Transfer
τ_L -Skin-Friction At The Left Wall
θ_0 -Mean Non-Dimensional Temperature

Superscripts:

* -Superscript For Dimensional Quantities

References:

1. Nakayama A, Koyama H (1991) Buoyancy induced flow of non-Newtonian fluids over a non-isothermal body of arbitrary shape in a fluid saturated porous medium. Appl Sci Res 48: 55–70
2. Ariel PD (1994) The flow of viscoelastic fluid past a porous plate. Acta Mech 107: 199-204
3. Andersson HI (1992) MHD flow of a viscoelastic fluid past a stretching surface. Acta Mech 95: 227–230
4. Pillai KMC, Sai KS, Swamy NS, Natraja HR, Tiwari SB, Rao BN (2004) Heat transfer in a viscoelastic boundary layer flow through porous medium. Comput Mech 34: 27-37
5. Sharma PR, Pareek D (2006) Unsteady flow and heat transfer through an elastico-viscous liquid along an infinite hot vertical porous moving plate with variable free stream and suction. Bull Cal Math Soc 98:97–108
6. Sharma PR, Sharma S (2005) Unsteady two dimensional flow and heat transfer through an elastico-viscous liquid along an infinite hot vertical porous surface bounded by porous medium. Bull Cal Math Soc 97:477–488
7. M.V. L. Bharathi, K. Jayalakshmi , (1, □)-Derivations On Lie Ideals In □ - Prime Near-Rings; Mathematical Sciences International Research Journal : Issn 2278-8697 Volume 6 Issue 1 (2017), Pg 22-25
8. Rahmann MM, Sarkar MSA (2004). Unsteady MHD flow of viscoelastic Oldroyd fluid under time varying body forces through a rectangular channel. Bulletin of Cacutta Mathematical Society 96: 463-470
9. Singh AK Singh NP (1966) MHD flow of a dusty viscoelastic liquid through a porous medium between two inclined parallel plates. Proc of National Academy of Sciences India 66A: 143-150
10. Datti PS, Prasael KV, Abel MS, Toshi M (2004) MHD viscoelastic fluid flow over anon-isothermal stretching sheet. Int J Eng Sci 42: 935–946
11. Roy JS, Chaudhury NK (1980) Heat transfer by laminar flow of an elastico-viscous liquid along a plane wall with periodic suction. Czech J Physics 30: 1199-1209
12. Hameed M, Nadeem S (2007) Unsteady MHD flow of a non-Newtonian fluid on a porous plate. J Math Anal Appl 325: 724–733
13. Hayat T, Asghar S, Siddiqui AM (1968) Periodic unsteady flows of a non-Newtonian fluid. Acta Mech 131: 169-173
14. Mujamil .M. Dakhani,.Zishan Ali .I. Dakhani, Solution To Deal With Research Interference In The Iot: Threats And Attacks On Iot And Its Resolution. ; Mathematical Sciences International Research Journal : Issn 2278-8697 Volume 6 Issue 1 (2017), Pg 26-30
15. Kumar B, Sivaraj R (2011) MHD mixed convective viscoelastic fluid flow in a permeable vertical channel with Dufour, effect and chemical reaction. Int J Appl Math 14:79–96
16. Singh KD (2012) Viscoelastic mixed convection MHD oscillatory flow through a porous medium filled in a vertical channel. Int J Physical and Mathematical Sciences 3: 194-205
17. Singh KD (2013) MHD mixed convection visco-elastic slip-flow through a porous medium in a vertical porous channel with thermal radiation. Kragujevac J Sci 35: 27-40
18. Attia HA (2004) Unsteady Hartmann flow of a viscoelastic fluid considering the Hall effect. Canadian J of Physics 82: 127
19. Attia HA (2010) Unsteady MHD Couette flow of a viscoelastic fluid with heat transfer. Kragujevac J Sci 32: 5-15
20. Alphonsa Mathew, Singh KD (2015) Hall current effect on radiating span-wise fluctuating MHD convective flow through porous medium in a vertical porous channel. Kragujevac Journal of Science 37: 49-63
21. Damseh RA, Shannak BA (2010) Visco-elastic fluid flow past an infinite vertical porous plate in the presence of first order chemical reaction. Appl Math Mech 31:955–962
22. K. Alagesan A. Charles Sagayaraj, M. Reni Sagayaraj, S. Anand Gnana Selvam, A Study On The Shortest Queueing Models With Parallel Queues; Mathematical Sciences International Research Journal : Issn 2278-8697 Volume 6 Issue 1 (2017), Pg 31-34

23. Sivaraj R, Rushi Kumar B (2012) MHD mixed convective flow of viscoelastic and viscous fluids in a vertical porous channel. Applications and Applied Mathematics 7: 99 – 116
24. Misra JC, Shit GC, Chandra S, Kundu PK (2011) Hydromagnetic flow and heat transfer of a second-grade viscoelastic fluid in a channel with oscillatory stretching walls: application to the dynamics of blood flow. J Eng Math 69: 91–100
25. P. G. Siddheshwar, T. N. Sakshath, Rayleigh- Bénard Convection Of Newtonian Nanoliquids In A Saturated, Rotating High Porous Medium; Mathematical Sciences International Research Journal : Issn 2278-8697 Volume 6 Issue 1 (2017), Pg 35-38
26. Choudhury R, Kumar Das S (2014) Visco-elastic MHD free convective flow through porous media in presence of radiation and chemical reaction with heat and mass Transfer. Journal of Applied Fluid Mechanics 7(4): 603-609
27. Madan Gopal Gorla, Khem Chand, Asha Singh (2016) Unsteady Heat and Mass Transfer in MHD Viscoelastic Fluid Flow through Porous Medium between Two Inclined Porous Parallel Plates with Soret Effect and G-Jitter Force. Journal of Indian Mathematical Society 83: 289-312
28. Walters K (1964) On second order effect in elasticity, plasticity and fluid dynamics. IUTAM Int. Symp. (Reiner, M. and Abir, D. eds) Pergamon Press, New York.
29. Attia, Hazem Ali, Karem Mahmoud Ewis (2010) Unsteady MHD Couette flow with heat transfer of a viscoelastic fluid under exponential decaying pressure gradient. Tamkang J Sci and Engng 13: 359-364
30. Kumar Rakesh, Khem Chand (2011) Effect of slip-condition and Hall current on unsteady MHD flow of a viscoelastic fluid past an infinite porous plate through porous medium. Int J of Engineering Science and Technology 3(4): 3124-3133
31. Nield DA, Bejan A (1998)Convection in Porous Media (2nd ed.)Springer-Verlag, Berlin

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